A Variable Speed Limit Controller for Recurrent Congestion Based on the Optimal Solution

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ABSTRACT

The main goal of this paper is the proposal and simulation of a SPEed limit controller for Recurrent Traffic jams (SPERT) that approximates the behavior of an optimal controller when congestion profiles are similar to the typical one. In order to achieve this goal, the optimal solution for the typical demand profile is computed and used as a first estimation for the logic-based controller. If the real congestion differs from the typical one, the values of the speed limits are adapted by advancing or delaying their activation and deactivation. The results show that the proposed controller is able to approach the optimal behavior while eliminating on-line computational cost and increasing robustness.

Keywords: Variable Speed Limits, Freeway Traffic Control

1 INTRODUCTION

Traffic congestion on freeways causes many social and economic problems like waste of time and fuel, a greater accident risk, and an increase in pollution. Much research has been focused on solving these problems in recent years. Since the construction of new freeways is not always a viable option or it is too costly, other solutions have to be found. In many cases, the use of dynamic control signals such as ramp metering, Variable Speed Limits (VSL), reversible lanes, and route guidance may be an economical and effective solution. Ramp metering and VSL have already been successfully implemented in practice in USA, Germany, Spain, Netherlands, and other countries (*1*).

When computing these control signals, the use of appropriate non-local and multivariable techniques can considerably improve the reduction in the total time spent by the drivers and other traffic performance indices like emissions or fuel consumption (*2*). Among the available options described in the literature, the methods based on Model Predictive Control (MPC) (*3*), which minimizes a cost function within a receding horizon approach, have shown to substantially improve the performance of the controlled traffic network in various simulation studies (*4–8*).

The main problem of MPC is that the computation time quickly increases with the size of the network, making it difficult to apply centralized MPC for large traffic networks. Distributed and hybrid techniques may relieve these limitations as can be seen in (*9*) and (*10*) but, unfortunately, the obtained controllers are still too complex to be implemented in real time for large networks and, moreover, they are not robust in case of communication or measurement failures. Therefore, completely centralized control of large networks is still viewed by most practitioners as impractical and unrealistic. In order to overcome this practical problem, easy-to-implement control algorithms have been designed for ramp metering (*11*) and reversible lanes (*12*). However, an easy-to-implement VSL control algorithm that approximates the performance of an MPC controller has to be necessarily a bit more complex that the ones proposed for ramp metering and reversible lanes.

In the literature, two practicaly implementable controllers designed to reduce congestion using VSL have been previously proposed and tested with successful results. In (*13*), a control algorithm (SPECIALIST) based on shock wave theory is proposed. This controller is able to solve/reduce isolated shock waves that do not necessarily always happen at the same time on different days (or they do not have the same magnitude). However, this controller does not take into account the optimal solution and, in some cases, solving a shock wave could create a new traffic jam or increase an existing one as can be seen in (*14*). In (*15*), a local VSL controller (Feedback mainstream traffic flow control or MTFC) is proposed that uses a cascade control structure with feedback of the density at the bottleneck area and the flow downstream the VSL application area. An extension of the Feedback MTFC in case of multiple bottlenecks is proposed in (*16*). However, similarly to SPECIALIST, this controller does not consider the optimal solution, entailing significant suboptimalities in some cases. For example, in (*16*) the TTS reduction is a 17.09% using a feedback controller and a 24.55% using an optimal controller.

When designing a VSL control, it has to be taken into account in general that a linear or logic-based controller for VSL, which can perform properly for one particular kind of congestion, is not going to approach the MPC behavior for other kinds of congestion.

Therefore, we propose the use of two control levels. In the upper level, a scheduling controller detects online the main kinds of congestion (recurrent congestion, shock waves, or unexpected capacity reductions) and, in the lower level, a practically implementable controller for each kind of congestion is used. This paper focuses on the lower-level VSL control algorithm for the first considered kind of congestion (recurrent congestion). The proposed controller is based on the optimal solution computed for the typical demand and can be applied in practice to large traffic networks.

Firstly, Section II introduces the macroscopic model used (METANET) and Section III summarizes the main aspects about computation of the optimal solution. Section IV explains the main characteristics of the proposed controller (SPERT: A SPEed limit controller for Recurrent Traffic jams), whose simulation results, for the hypothetical network presented in Section V, are shown in Section VI.

2 PREDICTION AND SIMULATION MODEL

In this work, the macroscopic traffic model METANET (*17*) has been selected for both simulation and control. Note, however, that the proposed controller could be tested and computed in a similar way using other macroscopic traffic models like CTM (*18*). METANET provides a good trade-off between simulation speed and accuracy and it can handle control actions such as ramp metering (*11*), route guidance(*19*), reversible lanes (*12*), and VSL (*4, 20*). The traffic network is represented as a graph where the links (indexed by m) correspond to freeway stretches. Each link m is divided into N_m segments of length L_m with λ_m lanes. Each segment i is dynamically characterized by the traffic density $\rho_{m,i}(k)$ and the mean speed $v_{m,i}(k)$ where k correspond to the time instant $t = kT$ and T is the simulation time step. For simplicity, in this paper all segment are considered to have different lengths and, therefore $N_m = 1 \ \forall \ m$, making it unnecessary to differenciate between links and segments; thus, hereafter only the index i will be used.

METANET uses two main equations describing the system dynamics. The first one expresses the conservation of vehicles:

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{\lambda_i L_i}(q_{i-1}(k) - q_i(k) + q_{\mathrm{r},i}(k) - \beta_i(k) q_{i-1}(k)) \tag{1}$$

where $q_{\mathrm{r},i}(k)$ is the traffic flow that enters the freeway from an on-ramp and $\beta_i(k)$ is the split ratio of an off-ramp (i.e. the percentage of vehicles exiting the freeway through an off-ramp in segment i). We set $\beta_i(k) = 0$ and $q_{r,i}(k) = 0$ for segments without an off-ramp or an on-ramp at the end of the segment, respectively. The traffic flow in each segment $q_i(k)$ can be computed for each time step using $q_i(k) = \lambda_i \rho_i(k) v_i(k)$.

The second equation expresses the mean speed as a sum of the previous mean speed, a relaxation term, a convection term, and an anticipation term:

$$v_i(k+1) = v_i(k) + \frac{T}{\tau_i}(V(k) - v_i(k)) + \frac{T}{L_i} v_i(k)(v_{i-1}(k) - v_i(k)) - \frac{\mu_i T}{\tau_i L_i} \frac{\rho_{i+1}(k) - \rho_i(k)}{\rho_i(k) + K_i} \tag{2}$$

where K_i, τ_i and μ_i are model parameters that have to be estimated for each segment and $V(k)$ is the desired speed by the drivers (3). As proposed in (*4*), the model can take different values for μ_i, depending on whether the downstream density is higher (μ_{H}) or lower (μ_{L}) than the density in the actual segment. The desired speed is modeled by the following equation which includes the effect of the VSL as in (*4*):

$$V(k) = \min(v_{\mathrm{f},i} e^{-\frac{1}{a_i}\left(\frac{\rho_i(k)}{\rho_{\mathrm{c},i}}\right)^{a_i}}, (1+\alpha) V_{\mathrm{c},i}(k)) \tag{3}$$

where α is a model parameter, $V_{\mathrm{c},i}(k)$ is the value of the VSL, a_i is a model parameter, $v_{\mathrm{f},i}$ is the free flow speed that the cars reach in steady state, and $\rho_{\mathrm{c},i}$ is the critical density (the density corresponding to the maximum flow in the fundamental diagram). In other references (*15*) VSL are included in the model by adapting the parameters of the fundamental diagram ($\rho_{\mathrm{c},i}$, $v_{\mathrm{f},i}$ and a_i).

An extra penalization term is added to the speed equation (2) if there is an on-ramp in order to account for the speed drop caused by merging phenomena:

$$-\frac{\delta_i T q_{\mathrm{r},i}(k) v_i(k)}{L_i \lambda_i (\rho_i(k) + K_i)} \tag{4}$$

where δ_i is a model parameter.

In order to complete the model, the following equation defines the flow that enters from an uncontrolled on-ramp.

$$q_{\mathrm{r},i}(k) = \min\left(C_{\mathrm{r},i}, D_i(k) + \frac{w_i(k)}{T}, C_{\mathrm{r},i}\frac{\rho_{\mathrm{m},i} - \rho_i(k)}{\rho_{\mathrm{m},i} - \rho_{\mathrm{c},i}}\right) \tag{5}$$

where $\rho_{\mathrm{m},i}$ and $C_{\mathrm{r},i}$ are model parameters and $w_i(k)$ is the queue lenght on a ramp on segment i, the dynamic of which are defined by:

$$w_i(k+1) = w_i(k) + T(D_i(k) - q_{\mathrm{r},i}(k)) \tag{6}$$

where $D_i(k)$ is the demand of the on-ramp connected to segment i. The mainline flow entering the first segment and the downstream density of the last segment are modeled as explained in (*4*).

3 OPTIMAL SOLUTION

The optimal solution for the typical demand is found by solving the following optimization problem with cost function $J(k)$ (see (8)), which is used to measure the performance of the system with respect to the VSL sequence:

$$\min_{V_{\mathrm{c,t}}(k)} J(k) \quad \text{with } V_{\mathrm{c},j}(k) \in S \tag{7}$$

where S is the set of allowed values for the VSL, $V_{\mathrm{c,t}}(k) = [V_{\mathrm{c},j_1}(k), V_{\mathrm{c},j_1}(k+1), ..., V_{\mathrm{c},j_1}(k+N_\mathrm{s}-1), V_{\mathrm{c},j_2}(k)$, $V_{\mathrm{c},j_2}(k+1), ..., V_{\mathrm{c},j_{N_\mathrm{VSL}}}(k+N_s-1)]$ is the vector contaning the VSL values, N_VSL the number of VSL gantries and N_s is the number of time steps. The cost function contains one term for the TTS, another term that limits (using a soft constraint) the maximum values of the queues, and a third term penalizing VSL variations:

$$J(k) = \sum_{\ell=1}^{N_s}[T\sum_{i\in \mathrm{O}} w_i(k+\ell) + T\sum_{i\in I}(\rho_i(k+\ell)L_i\lambda_i) + \sum_{i\in O}\Omega_i(k+\ell) + \psi\sum_{i=1}^{N_\mathrm{VSL}}(V_{\mathrm{c},i}(k+\ell) - V_{\mathrm{c},i}(k+\ell-1))^2] \tag{8}$$

where $\Omega_i(k+\ell)$ is a penalization term that is different to zero, and considerably larger than the other terms of the cost function, if the corresponding queue constraint is violated, O is the set of all the segments with an on-ramp, I is the set of all the segments, and ψ is a tuning parameter.

The optimization may been computed continuously using optimization algorithm RPROP (resilient backpropagation) (*21, 22*). Subsequently, the continuous VSL values have been discretized. It has to be pointed out that, in general, is necessary to run the algorithm many times (with different initial points) in order to avoid local minima. Another possibility is to directly optimize the VSL profiles by using discrete optimization (*10*). Hereafter, the optimal VSL profiles computed using the typical demand will be denoted by Nominal VSL.

4 SPERT: A SPEED LIMIT CONTROLLER FOR RECURRENT TRAFFIC JAMS

The employment of optimal control techniques in order to compute online the speed limit values is not deemed sufficiently practicable for ready field implementation because of the computation times required, the need of accurate calibrations and demand predictions, the presence of local minima, the need for robustness of the controller against communication or measurement failures, the counter-intuitive controller behavior, and other aspects.

Therefore, this paper proposes a simple yet efficient VSL control strategy that approximates the behavior of the optimal controller without need of any on-line optimization.

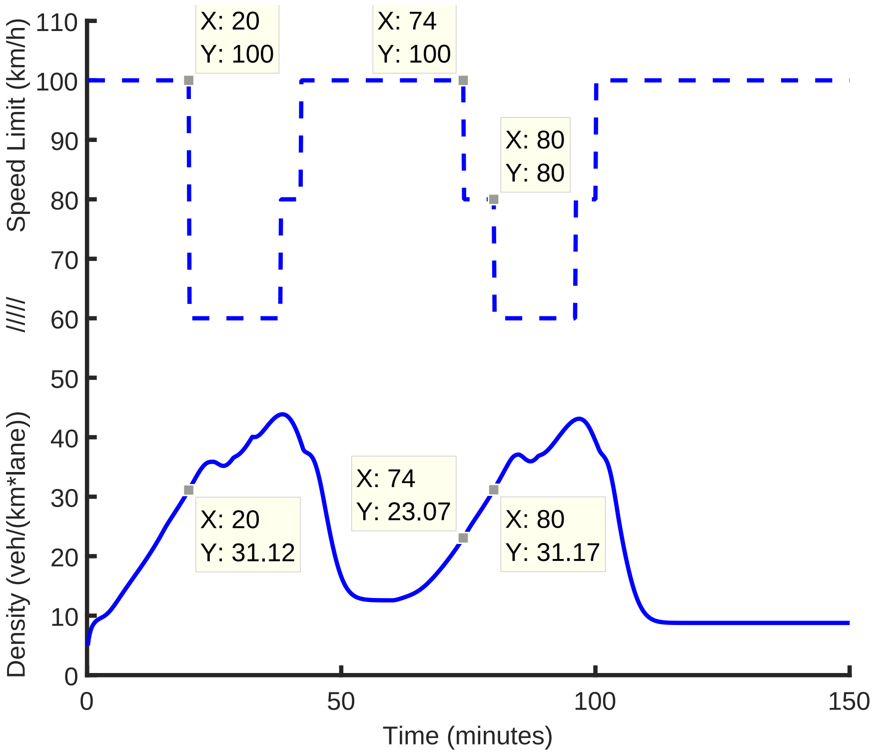

FIGURE 1 Bottleneck density and VSL of segment *i*.

The controller is designed for solving recurrent congestion caused by bottlenecks. Therefore, the controller will not be able to solve congestion caused by no-recurrent moving shock waves or unexpected capacity reductions. However, a large percentage of the congestion created in the freeways around cities is due to recurrent bottlenecks, which create similar congestion profiles for different days (*23*).

The algorithm is composed of the following steps. The first 4 steps are computed off-line so they only have to be done once and their computation load is not a limitation.

1. The typical demand is obtained by averaging the measured demands of weekdays with available measurements and without incidents. In the case of having different congestion/demand profiles depending on weather conditions or other measurable/estimable events, one typical demand should be defined for each case. For noisy typical demand is noisy, a smoother demand should be obtained by using a filter (for example, an Exponential Smoothing Filter (*24*)) in order to reduce the number of suboptimal local minima that may appear during the optimization process at step 2 and, if necessary, more advanced methods for demand estimation could be used (*25*).

2. The discrete optimal solution for the typical demands obtained in step 1 is computed off-line by optimizing the global network as explained in Section 3.

3. The different recurrent traffic jams appearing in the network are divided in time and space identifying the corresponding bottleneck segments. In the simulation done in this paper, there is only one bottleneck (on segment 9) and there are two main recurrent jams (one during the first hour and other one during the rest of the simulation). For large networks, this bottlenecks identification and the recurrent congestion splitting should be automatized. This will be the topic of a future paper.

4. The density thresholds that will determine when a VSL has to be increased or decreased are computed

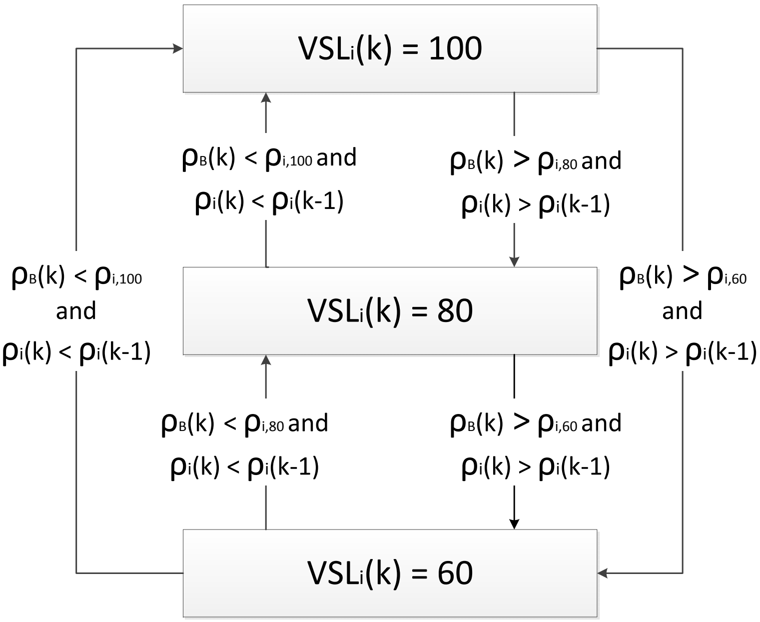

FIGURE 2 SPERT: Logic-based controller for VSL on segment *i* with three VSL values.

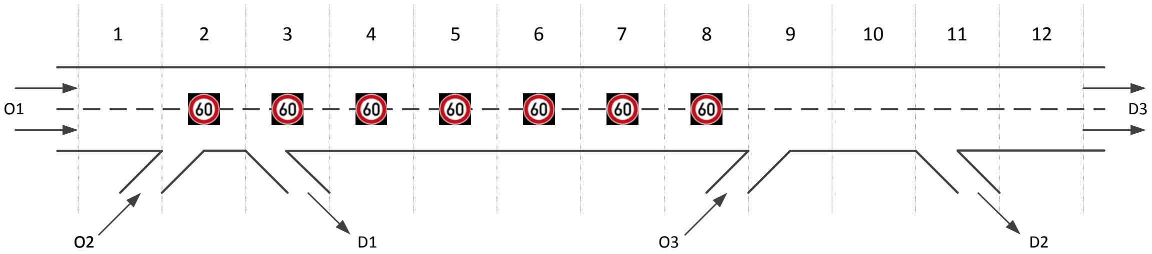

FIGURE 3 Simulated Network.

by analyzing the nominal simulation (the scenario with nominal demands and optimal VSLs): $\rho_{i,80}(k)$ is the density in the bottleneck segment at the time that the Nominal VSL on segment i is decreased to 80 km/h for the first time (for the considered traffic jam) using the typical demand. An example can be seen in Fig. 1, where it is shown the density of a bottleneck and the corresponding optimal VSL of segment i. When the Nominal VSL of segment i decreases from 100 km/h to 60 km/h in minute 20, the bottleneck density is 31.12 veh/ (km lane). Therefore, $\rho_{i,60}(k)$ for the first traffic jam (during the first hour) will be equal to 31.12 veh/(km lane). For the second traffic jam, the VSL of segment i is firstly decreased to 80 km/h in minute 74 and them, in minute 80, it is decreased to 60 km/h. Therefore, $\rho_{i,80}(k) = 23.07$ veh/(km lane) and $\rho_{i,60}(k) = 31.17$ veh/(km lane) for the second traffic jam. It has to be pointed out that, for clarity and simplicity, the algorithm has been defined in this example for three VSL values but it can be generalized for n VSL values.

5. The online controllers are implemented using the logic in Fig. 2 and the density thresholds computed in the previous step. SPERT activates and deactivates the corresponding variable speed limit when $\rho_{\mathrm{B},j}(k)$

(the densities of the bottlenecks affected by the corresponding speed limits $V_{c,i}(k)$) reaches the same value for which $V_{c,i}(k)$ was activated in the Nominal case. Moreover, in order to avoid undesirable oscillations of the speed limit values and, thus, density and speed oscillations, an additional constraint is included. This constraint only allows to increase the VSL when the bottleneck density is decreasing and vice versa. When dealing with noisy measurements, these densities have to be an aggregation of data during the last minutes. If desired, strong VSL variations can be bounded (specially for lowering VSLs) in order to increase safety because, for example, to decrease a VSL from 100 to 60 km/h in one step may be too abrupt for the drivers.

The main advantages of the proposed controller (SPERT) are:

- The online computation is almost instantaneous.
- The controller is implemented locally.
- The controller is based on the optimal solution outperforming other local controllers in situations where the global solution differs substantially from the local one as in (*2*).
- If a macroscopic model of the network is available or it can be automatically identified, the design process can be fully automatized; so a control law for a large real network could be obtained without any human intervention. It has to be pointed out that for quite large networks, the off-line computation of the Nominal VSL may take such a long time that, in these cases, distributed algorithms (*9*) or other kind of relaxation have to be employed.

The main disadvantage of the proposed controller is that it only works for congestion profiles relatively similar to the typical one. In case of unexpected congestion like accidents, non-recurrent shock waves coming from downstream segments, etc... other control algorithms should be used such as (*13, 15*). The triggering conditions defining when each controller has to be active or which typical demand has to be used (in case of being more than one) have to be implemented in a higher level controller.

5 SIMULATED NETWORK

A hypothetical 12 km long freeway stretch, shown in Fig. 3 has been used in order to simulate the proposed controllers. The freeway has $N = 12$ segments with a length of $L_i = 1000$ m and with $\lambda_i = 2$ lanes. There are 7 VSL (from segment 2 to segment 8), two on-ramps on segments 2 and 9 (uncontrolled) and two off-ramps on segments 3 and 11.

TABLE 1 METANET parameters

a	v_f	ρ_c	τ	μ_H
1.867	102 km/h	33.5 veh/(km lane)	18 s	20 km^2/h
μ_L	α	ρ_m	K	δ
60 km^2/h	0.1	180 veh/(km lane)	40	0.0122

All the METANET parameters (which can be seen in Table 1) are considered to be the same for all the segments. The simulation time chosen is two and half hour corresponding to 75 controller sample steps ($T_c = 120$ s) and 900 simulation steps ($T = 10$ s). The set of allowed VSL is $S = \{60, 80, 100\}$ km/h and no implementation constraints have been considered (i.e. the VSL are allowed to change directly in space and time from 60 km/h to 100 km/h and vice versa). The off-ramp split rates are considered constant

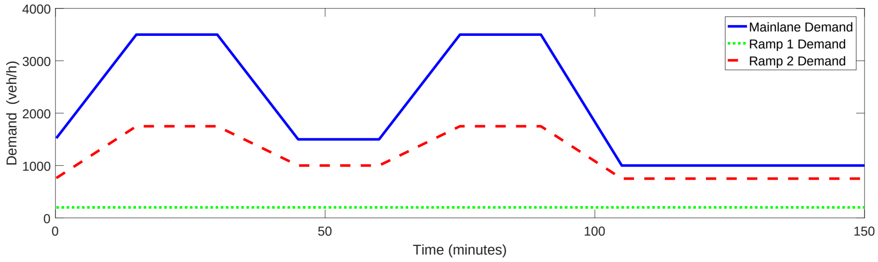

FIGURE 4 Typical Demands

and equal to the 20% of the traffic flow ($\beta_3(k) = \beta_{11}(k) = 0.2\ \forall\ k$) and the on-ramps have a capacity of $C_{r,2} = C_{r,9} = 2000$ veh/h.

The considered typical demand for the mainline and the on-ramps can be seen in Fig. 4. These demands reproduce two flow increases during two consecutive peaks hours (for example, 8 AM and 9 AM). Other 10 scenarios have been considered in order to test to proposed controller under different traffic conditions. These scenarios are obtained by increasing or decreasing (during the entire simulation) one of the demands (mainline or ramp 2) or the split ratios. The considered scenarios can be seen on Table 2.

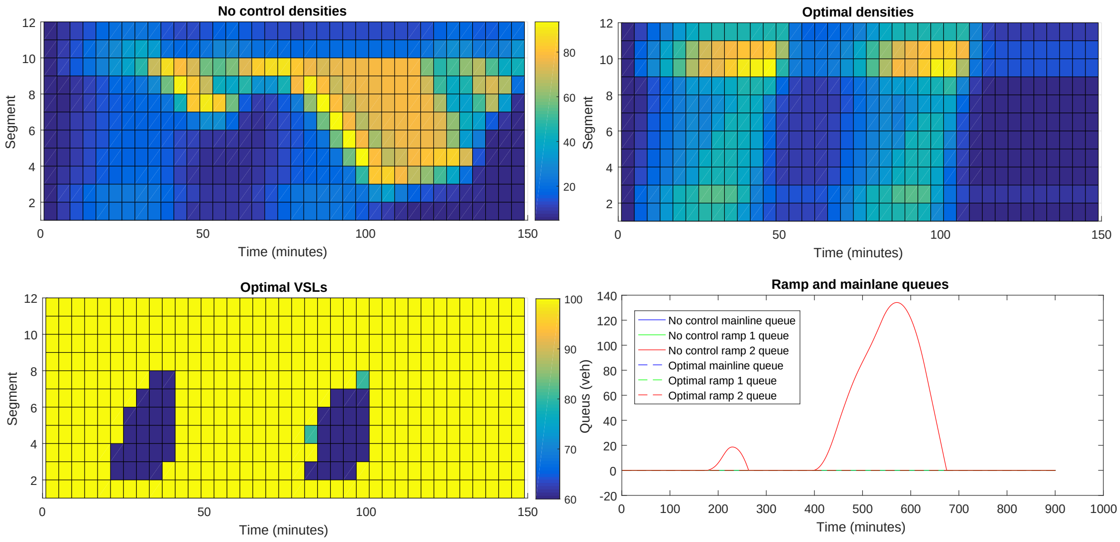

FIGURE 5 Densities, VSL and queues for no-control and Nominal VSL in Scenario 1

6 RESULTS

This section shows the main results obtained by simulation for the different scenarios and control algorithms. The optimizations have been computed continuously using RPROP and the results have been discretized by rounding. The numerical results are summarized in Table 2.

The no-control case simulation (i.e. with the VSL set equal to 100 km/h) entails a Total Time Spent (TTS) of 1783.3 veh h and congested density profiles as can be seen in Fig. 5.

In scenario 1 (typical demand), the nominal VSL reduces the TTS by 53.2% with respect to the no-control case by substantially decreasing the congestion and also removing the on-ramp queues as can be seen in Fig. 5.

TABLE 2 TTS Reduction (%)

	No Control	Nominal VSL	Optimal Controller	SPERT	Local MTFC
Scenario 1: Typical Demand	0% (1783.3 veh h)	53.2%	53.2%	52.8%	52.9%
Scenario 2: Mainstream Demand 10% Decreased	0% (710.6 veh h)	-2.5 %	0.1%	-0.3%	0%
Scenario 3: Mainstream Demand 10% Increased	0% (2731.1 veh h)	2.9%	11.8%	11.2%	7.9%
Scenario 4: 2nd Ramp Demand 10% Decreased	0% (790.1 veh h)	0.0%	1.3%	1.0%	1.1%
Scenario 5: 2nd Ramp Demand 10% Increased	0% (2636.3 veh h)	10.4%	19.9%	16.1%	14.1%
Scenario 6: Mainstream Demand 5% Decreased	0% (862.7 veh h)	10.3%	11.3%	11.0%	11.0%
Scenario 7: Mainstream Demand 5% Increased	0% (2323.4 veh h)	20.4%	53.7%	53.1%	44.4%
Scenario 8: Split Ratios 20% Increased	0% (785.2 veh h)	2.2%	3.5%	3.1%	3.3%
Scenario 9: Split Ratios 20% Decreased	0% (2379.5 veh h)	13.5%	54.0%	46.6%	43.4%
Scenario 10: 2nd Ramp Demand 10% Increased and Mainstream Demand 10% Decreased	0% (1883.2 veh h)	55.9%	56.2%	56.1%	56.1%
Scenario 11: Split Ratios 20% Decreased, Mainstream Demand 10% Decreased and 2nd Ramp Demand 10% Increased	0% (2523.1 veh h)	4.3%	22.5%	19.2%	17.1%
Mean TTS reduction	0%	15.5%	26.1%	24.5%	22.8%

However, it can be seen that Nominal VSL performs quite suboptimally when the traffic conditions differ from the optimized ones. For example, for Scenario 7, the TTS reduction is 20.4% versus 53.7% for the optimal case (computed using the real demands of the scenario) and, for, Scenario 9, the TTS reduction is 13.5% versus 54.0% for the optimal case.

SPERT shows a behavior closer to the optimal solution for all simulated scenarios. The biggest difference between the TTS obtained with SPERT and the minimum reachable TTS (optimal controller) is only 7.3%, in scenario 9. In fact, the observed behavior obtained with SPERT is almost equivalent to the optimal one. SPERT performs better than the nominal VSL for all the simulated scenarios. For Scenarios 3, 7 and 9, the improvement obtained with SPERT is especially significant compared to Nominal VSL (53.1% and 11.2% versus 20.4% and 2.9%, respectively).

In Scenario 2, the uncontrolled system only reaches congestion during a quite short period of time so the TTS cannot be reduced significantly (0.1% reduction). In this case, an incorrect use of the VSL could increase the TTS which is, obviously, not desirable. For example, using the Nominal VSL the TTS is increased with 2.5%. However, SPERT reacts to the decreased densities (compared with the nominal case), so the TTS is almost not increased (0.1% increase).

Finally, a comparison with the controller proposed in (*15*) (Local MTFC) is also included. The parameters of the controller have been optimized in order to maximize TTS reduction in Scenario 1. This controller also shows a good behavior in the remaining scenarios but with slightly worse performance than SPERT.

The mean TTS reductions obtained which each control algorithm match with the conclusions previously stated. As expected, the highest mean TTS reduction is obtained with the optimal controller (26.1%). The mean TTS reduction obtained with SPERT (24.5%) is slightly smaller than the optimal one, followed by the reduction obtained with MTFC (22.8%). On the other hand, Nominal VSL performs suboptimally with a mean TTS reduction of 15.5%.

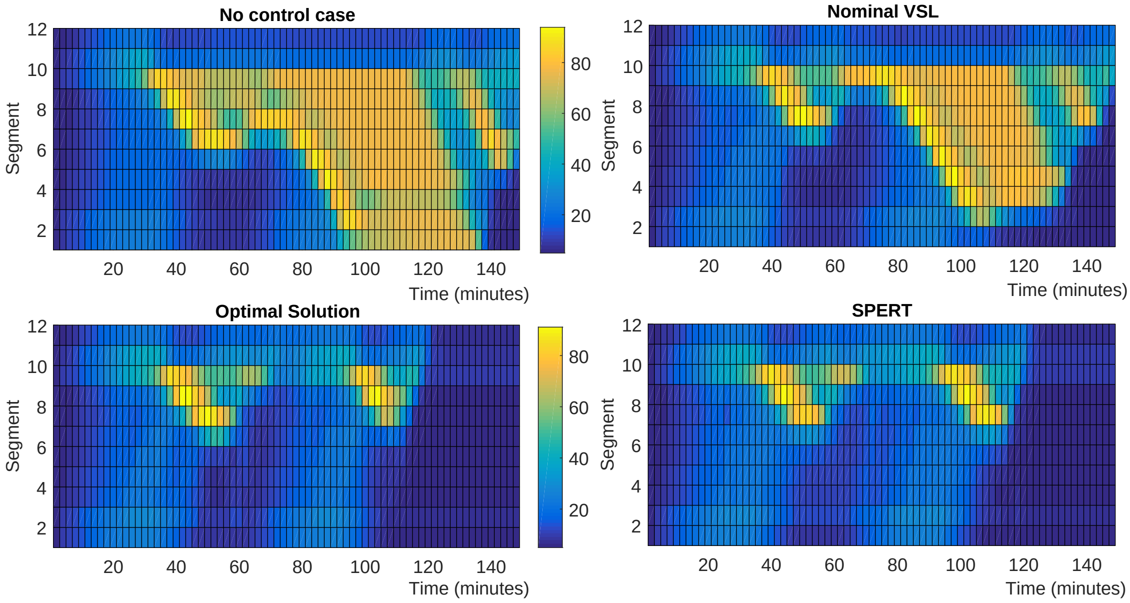

FIGURE 6 Density contour plots for Scenario 6

In Fig. 6, the density contour plots for Scenario 6 are shown. It can be observed that the density profiles obtained using SPERT are very similar to the optimal one, almost removing completely the traffic jam. On the other hand, the behavior observed using the Nominal VSL is not able to solve congestion during the second traffic jam.

7 CONCLUSIONS

This paper has proposed a control algorithm (SPERT) for Variable Speed Limits (VSL), based on the optimal solution in case of recurrent congestion, that can be applied in practice to large traffic networks. SPERT makes a trade-off between practical feasibility and optimality by combining advantages of optimal and easy-to-implement controllers.

The results show that an optimal controller for VSL performs quite suboptimally in scenarios that differ from the one used for optimization, even when the TTS is decreased for the majority of them. On the other hand, the results show that, for the studied scenarios, the controller proposed approaches the optimal behavior, substantially improving the performance of the off-line computed solution.

In future work, the proposed algorithm will be generalized for larger networks and integrated into the framework of a two-level controller.

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